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TRANSFORMATION THEORY FOR THE COMPRESSIBLE TURBULENT
BOUNDARY LAYER WITH ARBITRARY PRESSURE GRADIENT

by

John E. Lewis, Toshi Kubota, and Wilmot H. Webb

September 1968

Advanced Research Projects Agency
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FOREWORD

This research was supported by the Advanced Research Projects Agency of the Department of Defense and was monitored by the Space and Missile Systems Organization of the Air Force Systems Command, Norton Air Force Base, San Bernardino, California. The work reported here is submitted as a technical report on Task 7.4, Chemically Reacting Turbulent Boundary Layer, of Penetration Systems Studies, Contract F04701-68-C-0041.

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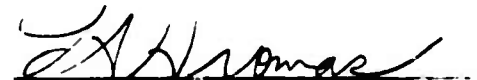The above statement is made for the reason that information in this report is embargoed under the Department of State International Traffic in Arms Regulations. This report may be released to foreign governments by departments or agencies of the U. S. government subject to approval of Space and Missile Systems Organization (SMYSE), Norton Air Force Base, California, or higher authority within the Department of the Air Force. Private individuals or firms require a Department of State export license.

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Major Walter D. McComb/SMYSE

ABSTRACT

We have reviewed Coles' analytical development of the transformation
and its application to the constant pressure turbulent boundary layer.
The limitations of the theory noted by Baronti and Libby are clarified,
and an attempt is made to explain the remaining discrepancies. The
equations governing the scaling functions (which link the high speed
boundary layer to an equivalent low speed flow) are derived in terms of
the integral properties of the low speed flow for an arbitrary pressure
distribution. The modeling is completed by coupling the pressure gradients
of the two flows and predicting the behavior of the integral properties
of the low speed flow. The resulting formulation predicts the integral
properties of the high speed turbulent boundary layer as well as the
required modeling to an equivalent low speed flow. Finally, the theory
is applied to a variety of situations.

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LIST OF ILLUSTRATIONS

A   Mach number parameter, $A = \dfrac{\frac{\gamma-1}{2} M_e^2}{1 + \frac{\gamma-1}{2} M_e^2}$

C_D Dissipation integral, $C_D = \displaystyle\int_0^\delta \dfrac{\tau}{\frac{1}{2}\rho_e u_e^2} \dfrac{\partial u/u_e}{\partial y} \, dy$

C_f Skin friction coefficient, $C_f = \dfrac{\tau_w}{\frac{1}{2}\rho_e u_e^2}$

$\overline{f}$ Low speed skin friction parameter, $\overline{f} = \dfrac{\overline{C}_f}{2}$

$\langle f \rangle$, $\langle f^2 \rangle$ Coles' sublayer constants, $\langle f \rangle = 17.2$
                                                   $\langle f^2 \rangle = 305$

G   Clauser's equilibrium parameter (see Appendix)

H   Form factor, $H = \theta/\delta^*$

$\mathcal{J}$ Viscous dissipation parameter, $\mathcal{J} = \left\{ \dfrac{\overline{R}_\theta}{R_\theta} \left(\dfrac{T_w}{T_\infty}\right)^3 \right\}^{1/2}$

k   Karman's constant, $k = .40$

M   Mach number

m   Mach number parameter, $m = \dfrac{\gamma-1}{2} M^2$

p	Static pressure
R_L	Reynolds number, $R_L = \dfrac{\rho_\infty u_\infty L}{\mu_\infty}$
T	Static temperature
T_o	Total temperature
T'	Ratio of turbulent to laminar shear stress, $(T' = \tau_t/\tau_\ell)$
u	Velocity component parallel to free stream
u_τ	Friction velocity, $u_\tau = \sqrt{\dfrac{\tau_w}{\rho_w}}$
v	Velocity component normal to free stream
x	Coordinate parallel to free stream
y	Coordinate normal to free stream
$\bar{\beta}_T$	Clauser's pressure gradient parameter, $\bar{\beta}_T = \dfrac{\bar{\delta}^*}{\bar{\tau}_w} \dfrac{d\bar{p}}{d\bar{x}}$
γ	Ratio of specific heats, $\gamma = c_p/c_v$
δ	Boundary layer thickness

δ^* Displacement thickness, $\delta^* = \displaystyle\int_0^\delta \left(1 - \dfrac{\rho u}{\rho_e u_e}\right)\, dy$

η, η' Scaling function

θ Momentum thickness, $\theta = \displaystyle\int_0^\delta \left(1 - \dfrac{u}{u_e}\right)\dfrac{\rho u}{\rho_e u_e}\, dy$

μ Viscosity

ν Kinematic viscosity, $(\nu = \mu/\rho)$

ξ, ξ' Scaling function

$\tilde{\pi}$ Low speed velocity profile parameter

ρ Density

σ, σ' Scaling function

τ Shear stress

ψ Stream function

Superscripts and Subscripts

$(\ )_\infty$ Refers to upstream infinity

$(\ )_e$ Refers to boundary layer edge

$(\ )_w$ Refers to wall condition

$\overline{(\ )}$ Refers to low speed flow

$(\ )_t$ Refers to turbulent flow

$(\ )_\ell$ Refers to laminar flow

1. INTRODUCTION

The dynamic equations governing the boundary layer are identical
in form for laminar and turbulent flow with the exception of the specific
form for the shear stress. While the simple Newtonian stress relation-
ship is valid everywhere for laminar flow, it is known to be valid only
very near the wall in a turbulent boundary layer. The prediction of the
turbulent boundary layer is, of course, much more difficult because of
the lack of understanding of the turbulent momentum transport. Most
analytical models rely on a simple gradient diffusion process (eddy
viscosity); and while such a method is perhaps adequate for low speed
flows, the effect of compressibility on such a model is unknown. This
difficulty, in addition to the existence of a vast amount of empirical
knowledge about the low speed turbulent boundary layer, makes the prospect
of using a mathematical transformation to account for the effects of
compressibility particularly inviting.

The objective of the present study is to develop a transformation
theory which will model a given supersonic turbulent boundary layer flow
of arbitrary pressure gradient $[u_e(R_x)$ - specified] into an equivalent
low speed boundary layer flow $[\bar{u}_e(\bar{R}_x)$ - predicted]. It will be found
that in developing this modeling, the integral properties of both the
high speed and the equivalent low speed flow will be predicted as well.

We begin by reviewing Coles'[1] analytical development of the
transformation and its application to the constant pressure turbulent
boundary layer. The inaccuracies of the theory noted by Baronti and
Libby[2] are clarified, and an attempt is made to explain the remaining
discrepancies.

The equations governing the scaling functions (which link the high
speed boundary layer to an equivalent low speed flow) are derived in
terms of the integral properties of the low speed flow for an arbitratry

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pressure distribution. The modeling is completed by relating the pressure gradients of the two flows and predicting the behavior of the integral properties of the low speed flow. The resulting formulation predicts the integral properties of the high speed turbulent boundary layer as well as the required modeling to an equivalent low speed flow.

Finally, the theory is applied to a variety of situations.

2. REVIEW OF COLES' TRANSFORMATION WITH CONSTANT PRESSURE APPLICATIONS

The use of a mathematical transformation to account for the effect of compressibility has been extensively applied to laminar boundary layers and wakes. Howarth,[3] Stewartson,[4] and many others have used this technique. Coles has formulated perhaps the most general transformation yet considered for the dynamic boundary layer equations. These equations for the steady, two-dimensional mean flow of a compressible fluid are

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \qquad (1)$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial \tau}{\partial y} \qquad (2)$$

The transformation,[*] simply stated, specifies the correspondence between a flow governed by these equations and an incompressible flow for which

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0 \qquad (3)$$

$$\overline{\rho}\, \overline{u}\, \frac{\partial \overline{u}}{\partial \overline{x}} + \overline{\rho}\, \overline{v}\, \frac{\partial \overline{u}}{\partial \overline{y}} = -\frac{d\overline{p}}{d\overline{x}} + \frac{\partial \overline{\tau}}{\partial \overline{y}} \qquad (4)$$

and $\overline{\rho}$ is constant.

The transformation set forth by Coles is represented by three initially unspecified scaling functions:

$$\frac{\overline{\psi}}{\psi} = \sigma \qquad (5)$$

---

[*]The term transformation will be used here in the same sense used by Coles, i.e., as a true mapping of one flow field into another rather than simply a mathematical manipulation.

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$$\frac{d\bar{x}}{dx} = \xi \tag{6}$$

$$\frac{\bar{\rho}}{\rho} \frac{\partial \bar{y}}{\partial y} = \eta \tag{7}$$

In order to insure the boundedness of the pressure gradient throughout each flow field, Coles imposes the condition that these functions be independent of y, i.e., $\sigma = \sigma(x)$, $\eta = \eta(x)$ and $\xi = \xi(x)$. Before pursuing this development, it is appropriate to comment on the ability of the transformation to predict the behavior of high Mach number boundary layers at constant pressure.

Baronti and Libby in attempting to evaluate the transformation at nominally constant pressure noted a systematic deviation with increasing Mach number in the "wake" region of the boundary layer but concluded that the "law of the wall region" was well predicted.

They arrive at this conclusion by choosing a value of $\bar{C}_f$ which allows the best agreement of the respective low speed and high speed velocity profiles in the inner logarithmic region of the boundary layer. However, this procedure precludes a test of the transformation in the wall region of the boundary layer since it forces a match there; in addition, it does not, in general, satisfy an essential requirement of the transformation which relates one station to another in the high speed and low speed flows, i.e., Coles' "law of corresponding stations", which for $\mu \sim T$ is simply $C_f R_\theta = \bar{C}_f \bar{R}_{\bar{\theta}}$. An additional but less fundamental point is that the choice by Baronti and Libby of δ, the boundary layer thickness, for normalization gives rise to uncertainties which can be large particularly for turbulent boundary layers where the edge is not well defined.

Using the law of corresponding stations, $C_f R_\theta = \bar{C}_f \bar{R}_{\bar{\theta}}$, and comparing the velocity profiles in the coordinates

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$$\frac{u}{u_e} = \frac{\bar{u}}{\bar{u}_e} \quad \text{vs.} \quad \frac{\bar{y}}{\bar{\theta}} = \int_0^y \frac{\rho}{\rho_e} \frac{dy}{\theta}$$

we eliminate both of the above objections to the Baronti-Libby comparison and the need to specify any of the scaling functions, hence, the need to resort to any ad hoc theory such as the "sublayer" or "substructure" hypothesis.

The results of applying the transformation in this way to the experimental data of Coles[5] and Korkegi[6] (adiabatic, flat plate) is shown in Figure 1 contrasted with the low speed data of Wieghardt[7] and the analytical form suggested by Coles.[1]* As can be seen, there is indeed a deviation from the low speed profiles which increases as Mach number increases. However, the deviation is seen to be more significant in the inner region than the outer region of the boundary layer which is contrary to the conclusions of Baronti and Libby. In addition, the thickness of the boundary layer is increased. The actual magnitude of the velocity in the wake region is within 2-3% of the predicted value whereas the deviation at $\bar{y}/\bar{\theta} = 1$ is seen to be as large as 10%.

Apparently, then, the transformation fails in the inner "law of the wall" region of the boundary layer to an extent which increases systematically with Mach number. Since this region typically represents less than 20% of the boundary layer thickness and it can reasonably be argued that the resulting error in overall (e.g., integral) properties of the layer may not be excessive, nevertheless the fundamental idea of Coles that a simple geometric mapping of a low speed to a high speed flow can account for effects of compressibility does not appear to be valid in detail. One phenomenon which is not expected to be accounted for by simple transformation theory is the increasing magnitude of sound pressure level [the irrotational fluctuation mode in the parlance of Kovasznay[8] and Morkovin[9]] with

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*The analytical form has the ambiguity previously mentioned regarding $\bar{\delta}$; however, here it is confined to an analytical form and affects only one of the two flows.

Mach number. Whether this phenomenon is important to the momentum exchange is not clear [see Schubert and Corcos[10]], but it is difficult to see how the complicated sound radiation field generated at high Mach number in the boundary layer could be accounted for merely by a mathematical transformation. Another phenomenon introduced by high Mach number is the variation of molecular transport coefficients (viscosity, conductivity) due to temperature variations in the boundary layer. An important process affecting turbulent momentum exchange in the region adjacent to the laminar sublayer is the "damping" of velocity fluctuations by viscous forces there. For an adiabatic supersonic boundary layer, the temperature near the wall is significantly higher than ambient and an increased dissipation of turbulent velocity fluctuations due to the increased viscosity is therefore expected in this region. This increase in viscous dissipation is believed to be a plausible explanation for the deviation of the transformed profiles in the law of the wall region. To give some quantitative support to this conjecture, we have made a rough estimate of this effect in the following way. Webb[11] has arrived at an empirical estimate of the effect of dissipation based on a simple gradient diffusion model which seems to give an accurate representation of the turbulent shear stress near the wall in confined low speed turbulent flows. This estimate is obtained by applying the known solution for the decay of velocity in a finite viscous vortex to the estimate of the damping of the turbulent velocity fluctuations which are convected during the mixing process. The result is that the local shear stress is reduced by a factor dependent on the local ratio of turbulent to laminar shear stress, $T' = \tau_t / \tau_\ell$. Figure 2 shows the change in velocity profiles for low speed pipe flow which results from arbitrarily increasing the value of the dissipation factor by various amounts. Qualitatively, the change appears similar to that due to increasing Mach number in the compressible adiabatic boundary layer.

A typical value of the ratio T' , evaluated at $y = \theta$ is estimated for a compressible boundary layer as follows:

$$T' \sim \frac{\rho \; \theta^2 (\partial u / \partial y)^2}{\mu \; \partial u / \partial y} \sim \frac{\theta^2}{\nu_w} \; (u_\tau / \theta) \sim \frac{\theta}{\nu_w} \sqrt{\frac{\tau_w}{\rho_w}} \sim R_\theta \sqrt{C_f} \; \frac{\mu_e}{\mu_w} \left(\frac{\rho_w}{\rho_e} \right)^{1/2}$$

Taking $\rho \sim T^{-1}$ and $\mu \sim T$ and recalling that $C_f R_\theta = \overline{C}_f \overline{R}_{\overline{\theta}}$, then the ratio of T for low speed flow to its value for compressible flow is

$$\mathcal{T} = \frac{\overline{T'}}{T'} = \left\{ \frac{\overline{R}_\theta}{R_\theta} \left(\frac{T_w}{T_e} \right)^3 \right\}^{1/2}$$

To see whether the parameter $\mathcal{T}$ indeed collapses the velocity profiles for various Mach numbers and Reynolds numbers, data taken by Coles[5] and by Lobb, Winkler and Persch[12] over a range of M_∞ up to 8.2 (including heat transfer) for $\mathcal{T} \simeq 6$ are shown in Figure 3. The remarkable agreement of these profiles for a fixed value of $\mathcal{T}$ strongly supports the offered explanation for the failure of the transformation theory near the wall. As an alternate display of the correlation, the maximum deviation of the experimental profiles from the transformation theory is plotted directly against the parameter $\mathcal{T}$ in Figure 4. Shown for comparison is the calculated deviation for pipe flow referred to earlier.

On the basis of these correlations, we tentatively conclude that viscous dissipation effects are responsible for the deviation of the transformation theory near the wall and, at least in principle, the transformation theory is invalid. However, for not excessively high Mach numbers, the deviation is small, it extends over only a small fraction of the boundary layer, and it does not preclude the application of the theory to obtain overall results.

3. GENERALIZATION TO VARIABLE PRESSURE ADIABATIC WALL

3.1 SCALING FUNCTIONS

For the constant pressure turbulent boundary layer, Coles derives the following relationships for the scaling functions σ, η, ξ :

$$\frac{\sigma}{\eta} = \frac{\overline{u}_{\infty}}{u_{\infty}} \quad \text{(constant)} \tag{8}$$

$$\frac{\xi}{\eta} = \frac{\rho_w \mu_w}{\overline{\rho}\ \overline{\mu}} \frac{d(\sigma\theta)}{d\theta} \tag{9}$$

$$\frac{\overline{\mu}}{\sigma \mu_{\infty}} = \frac{T_w}{T_{\infty}} - \langle f \rangle \frac{T_w - T_{0_{\infty}}}{T_{\infty}} \sqrt{\frac{\overline{C}_f}{2}} - \langle f^2 \rangle m_{\infty} \frac{\overline{C}_f}{2} \tag{10}$$

$$(\mu/T = \text{constant})$$

He arrives at these equations by invoking the condition of constant pressure, Newtonian friction at the wall, and an assumption known as the "substructure hypothesis" (which is similar to the constancy of the sublayer Reynolds number). The analysis is well detailed in Coles report and will not be repeated here.

If we take the substructure hypothesis to be independent of pressure gradient and heat transfer [see Coles[1]], we need only generalize equations 8 and 9. This assumption has been made in the following developments. While plausible, it is experimentally unverified.

The pressure gradients of the two flows are related[1] by

$$\frac{d\overline{p}}{d\overline{x}} = \frac{\overline{\rho}\ \sigma^2}{\xi\ \eta^2} \left(\frac{1}{\rho_e} \frac{dp}{dx} + \frac{u_e^2}{\eta/\sigma} \frac{d(\eta/\sigma)}{dx} \right) \tag{11}$$

This equation is one of the required relationships; however, it is in a

form which is not useful for our purposes in that it involves both $d\bar{p}/d\bar{x}$ and dp/dx, one of which is unknown _a priori_. However, it can be shown that by noting the behavior of the velocity in the vicinity of the wall [see Lewis[13]] the pressure gradients are linked by the following relationship:

$$\frac{\rho_e}{\rho_w} \frac{\theta}{\tau_w} \frac{dp}{dx} = \frac{\bar{\theta}}{\bar{\tau}_w} \frac{d\bar{p}}{d\bar{x}} \tag{12}$$

Combining equations 11 and 12 after some manipulation, we find that

$$\frac{1}{\eta/\sigma} \frac{d\eta/\sigma}{d\bar{x}} = \left\{ 1 - \frac{\rho_w}{\rho_e} \frac{\rho_w}{\bar{\rho}} \frac{\mu_w}{\bar{\mu}} \frac{\sigma\eta}{\xi} \right\} \frac{1}{\bar{\rho} \, \bar{u}_e^2} \frac{d\bar{p}}{d\bar{x}} \tag{13}$$

where, for reasons which will become obvious later, σ, η, ξ are taken as functions of $\bar{x}$ rather than x. Equation 13 replaces the constant pressure condition $\eta/\sigma = (u_\infty / \bar{u}_\infty)$ (constant).

One more equation is required and for this we utilize the relationship between the shear stress terms of the momentum equation,[1]

$$\frac{\partial \bar{\tau}}{\partial \bar{y}} = \frac{\bar{\rho}}{\xi} \frac{\sigma^2}{\eta^2} \left[\frac{1}{\rho} \left(\frac{\partial \tau}{\partial y} - \frac{\psi}{\sigma} \frac{\partial u}{\partial y} \frac{d\sigma}{dx} \right) + \frac{dp}{dx} \left(\frac{1}{\rho_e} - \frac{1}{\rho} \right) + \frac{1}{\eta/\sigma} \frac{d\eta/\sigma}{dx} \left(u_e^2 - u^2 \right) \right] \tag{14}$$

Integrating across the boundary layer, we find that

$$\frac{\xi\eta}{\sigma^2} \bar{\tau}_w = \tau_w + \frac{1}{\sigma} \frac{d\sigma}{dx} \cdot \int_0^\delta \psi \frac{\partial u}{\partial y} \, dy$$

$$+ \frac{dp}{dx} \cdot \int_0^\delta (1 - \rho/\rho_e) \, dy - \frac{1}{\eta/\sigma} \frac{d\eta/\sigma}{dx} \cdot \rho_e u_e^2 \cdot \int_0^\delta (1 - u^2/u_e^2) \frac{\rho}{\rho_e} \, dy$$

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Noting that

$$\int_0^0 (1 - u^2/u_e^2) \frac{\rho}{\rho_e} \, dy = \frac{\bar{\rho}}{\rho_e \eta} (\bar{\delta}^* + \bar{\theta})$$

and evaluating the integral, $\int_0^\delta (1 - \rho/\rho_e) \, dy$ by approximating the temperature field by the equation $(T_o/T_w) - 1 \simeq \frac{u}{u_e} [(T_{o_\infty}/T_w) - 1]$ we arrive at the required relationship

$$\frac{\rho_w u_w}{\bar{\rho} \bar{\mu}} \frac{\partial \eta}{\xi} = \frac{\bar{C}_f/2 - (\bar{\theta}/\rho) \frac{d\bar{u}}{d\bar{x}} + \frac{\bar{\delta}^*}{\bar{\rho} \bar{u}_e^2} \frac{d\bar{p}}{d\bar{x}} (1 + \bar{H})}{\bar{C}_f/2 + \left(1 + \frac{T_{o_\infty}}{T_w} \bar{H}\right) \frac{\bar{\delta}^*}{\bar{\rho} \bar{u}_e^2} \frac{d\bar{p}}{d\bar{x}}} \tag{15}$$

We have for convenience redefined the scaling functions in the following manner:

$$\zeta = \frac{\bar{\mu}}{\mu_x} \sigma' \qquad\qquad \eta/\sigma = \frac{\upsilon}{u} : '/\upsilon'$$

$$\zeta = \frac{\rho_\infty u_\infty/\mu_\infty}{\bar{\rho} \bar{u}_\infty/\bar{\mu}} \xi' \tag{16}$$

The resulting equations become

$$\frac{1}{\sigma'} = (1 + m_e) \left[\frac{T_w}{T_{o_\infty}} - \langle f \rangle \left(\frac{T_w}{T_{o_\infty}} - 1\right) \bar{f} - \langle f^2 \rangle \frac{m_e}{1 + m_e} \bar{f}^2\right] \tag{17}$$

$$\frac{1}{\eta'/\sigma'} \frac{d\eta'/\sigma'}{d\bar{R}_x} = \left\{\frac{T_{o_\infty}}{T_w} (1 + m_e)^{-1} \frac{P_e}{P_\infty} \frac{\sigma'\eta'}{\xi'} - 1\right\} \frac{1}{\bar{u}_e} \frac{d\bar{u}_e}{d\bar{R}_x} \tag{18}$$

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$$\frac{p_e}{p_\infty} \frac{\sigma' \eta'}{\xi'} = \frac{\bar{f}^2 - \frac{\bar{R}_\theta}{\sigma'} \frac{d\sigma'}{d\bar{R}_x} - (1 + \bar{H}) \frac{\bar{R}_\delta^*}{\bar{u}_e} \frac{d\bar{u}_e}{d\bar{R}_x}}{\bar{f}^2 - \left(1 + \frac{T_{o_\infty}}{T_w} \bar{H}\right) \frac{\bar{R}_\delta^*}{\bar{u}_e} \frac{d\bar{u}_e}{d\bar{R}_x}} \qquad (19)$$

where $\dfrac{p_e}{p_\infty} = \left\{1 - m_\infty \left(\dfrac{u_e^2}{u_\infty^2} - 1\right)\right\}^{\gamma/(\gamma-1)}$, $\dfrac{u_e}{u_\infty} = \eta'/\sigma' \dfrac{\bar{u}_e}{\bar{u}_\infty}$, $\bar{f} = \sqrt{\dfrac{\bar{C}_f}{2}}$

and $m_e = \dfrac{A}{1 - A}$, $A = \dfrac{m_\infty}{1 + m_\infty} (\eta'/\sigma')^2 \left(\dfrac{\bar{u}_e}{\bar{u}_\infty}\right)^2 = \dfrac{m_\infty}{1 + m_\infty} \left(\dfrac{u_e}{u_\infty}\right)^2$

Hence, given a low speed flow which is completely specified (at least insofar as its integral properties are concerned) these three relationships can be used to construct an equivalent supersonic flow for a given M_∞ .

The objective of the study is, of course, to take a specified velocity distribution in the supersonic flow, $u_e(R_x)$, and predict the necessary $\bar{u}_e(\bar{R}_x)$ to produce its low speed equivalent.

Returning to equation (12), it can be shown that

$$\frac{1}{\bar{u}_e} \frac{d\bar{u}_e}{d\bar{R}_x} = \frac{T_w}{T_{o_\infty}} (1 + m_e) \left(\frac{p_e}{p_\infty}\right)^{-1} \frac{1}{\sigma' \eta'} \frac{1}{u_e} \frac{du_e}{dR_x} \qquad (20)$$

Hence, given $u_e(R_x)$ we have an equation to make such a prediction provided that σ' and η' are known. These functions can be predicted by simply calculating the integral properties of the low speed flow simultaneously. Coupling such a calculation with the previously derived transformation laws, we have then arrived at a self-contained set of equations which predicts the modeling we seek. The integral properties of the supersonic turbulent boundary layer are also obtained.

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Returning to the scaling functions, we note that equation 19 requires $d\sigma'/d\overline{R}_x$, and hence for simplicity we have treated σ' as one of the dependent variables of the final system of ordinary differential equations. We differentiate equation 17 which becomes

$$\frac{2<f^2>\,A\,\overline{f}^2}{\eta'}\frac{d\eta'}{d\overline{R}_x} - \left[\frac{1}{\sigma'^2}(1 + m_\infty) + \frac{2<f^2>\,A\,\overline{f}^2}{\sigma'}\right]\frac{d\sigma'}{d\overline{R}_x}$$

$$+ \left[<f>\left(\frac{T_w}{T_{0_\infty}} - 1\right) + 2<f^2>\,A\,\overline{f}\right]\frac{d\overline{f}}{d\overline{R}_x} + 2A<f^2>\,\overline{f}^2\,\frac{1}{\overline{u}_e}\frac{d\overline{u}_e}{d\overline{R}_x} = 0 \qquad (21)$$

Combining equations (18) and (19)

$$\left[\overline{f}^2 - \left(1 + \frac{T_{0_\infty}}{T_w}\overline{H}\right)\frac{\overline{R}_\delta^*}{\overline{u}_e}\frac{d\overline{u}_e}{d\overline{R}_x}\right]\frac{1}{\eta'}\frac{d\eta'}{d\overline{R}_x}$$

$$-\left[\overline{f}^2 - \left(1 + \frac{T_{0_\infty}}{T_w}\overline{H}\cdot\frac{2 + m_e}{1 + m_e}\right)\frac{\overline{R}_\delta^*}{\overline{u}_e}\frac{d\overline{u}_e}{d\overline{R}_x}\right]\frac{1}{\sigma'}\frac{d\sigma'}{d\overline{R}_x}$$

$$= \left\{\left(\frac{T_{0_\infty}}{T_w}\frac{1}{(1 + m_e)} - 1\right)\overline{f}^2 + \left[\left(1 - \frac{T_{0_\infty}}{T_w}\frac{1}{1 + m_e}\right)\right.\right.$$

$$\left.\left. + \frac{T_{0_\infty}}{T_w}\overline{H}\frac{m_e}{1 + m_e}\right]\frac{\overline{R}_\delta^*}{\overline{u}_e}\frac{d\overline{u}_e}{d\overline{R}_x}\right\}\frac{1}{\overline{u}_e}\frac{d\overline{u}_e}{d\overline{R}_x} \qquad (22)$$

and recalling $\dfrac{dR_x}{d\overline{R}_x} = \dfrac{1}{\xi'}$, it follows from equation (19) that

$$\frac{dR_x}{d\overline{R}_x} = \left(\frac{p_e}{p_\infty}\sigma'\eta'\right)^{-1}\frac{\overline{f}^2 - \overline{H}\frac{\overline{R}_\delta^*}{\sigma'}\frac{d\sigma'}{d\overline{R}_x} - (1 + \overline{H})\frac{\overline{R}_\delta^*}{\overline{u}_e}\frac{d\overline{u}_e}{d\overline{R}_x}}{\overline{f}^2 - \left(1 + \frac{T_{0_\infty}}{T_w}\overline{H}\right)\frac{\overline{R}_\delta^*}{\overline{u}_e}\frac{d\overline{u}_e}{d\overline{R}_x}} \qquad (23)$$

Equations (20)-(23) and the integral formulation of the low speed flow (see Appendix) represent the total formulation which is composed of

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seven ordinary differential equations with seven dependent variables; $\overline{R_\delta^*}$, $\widetilde{\pi}$, $\overline{f}$, $\overline{u_e}/\overline{u}_\infty$, σ', η', R_x with $\overline{R_x}$ as the independent variable. These equations are well defined provided $u_e(R_x)$ is specified, and the variables are initialized.

We could, of course, resort to the particular high speed experiment of interest for such initial conditions. However, in order for the procedure to be useful in practice (i.e., general engineering predictions) it is necessary to choose these values independently of a particular experiment. We have chosen the values tabulated by Coles[1] for which the wake of the incompressible boundary layer disappears as being characteristic of the low speed turbulent boundary layer near its "origin".

$$\widetilde{\pi}_o = 0$$

$$\overline{C_f}_o = 5.9 \times 10^{-3}$$

$$\overline{R_\theta}_o = 4.25.$$

At the "origin" we have taken $R_{x_o} = \overline{R_{x_o}} = 0$ and $(\overline{u_e}/\overline{u}_\infty)_o = 1$; σ_o' can be calculated from equation (17), $M_{e_o} = M_\infty$ and, of course, $\eta_o' = \sigma_o'$.

The wall temperature, T_w/T_{o_∞}, has been carried throughout the analysis as arbitrary and hence formally non-adiabatic boundary layers could also be predicted. In a strict sense, however, Crocco's[14] suggestion that the heat transfer of the two flows must match can be shown to be a requirement of the mapping [see Lewis[13]]. In particular we must have

$$\frac{\theta}{T_e} \left(\frac{\partial T}{\partial y} \right)_w = \frac{\overline{\theta}}{\overline{T}_e} \left(\frac{\partial \overline{T}}{\partial y} \right)_w$$

and hence the low speed flow, while incompressible, cannot be of constant density if it is to correspond to a non-adiabatic compressible flow.

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4. APPLICATION OF THEORY

4.1 LOW SPEED - CONSTANT PRESSURE

In order to evaluate the formulation for the low speed turbulent boundary layer, particularly in light of our choice of initial conditions, we have made comparisons with the experimental data of Wieghardt[7] (Figure 5). $\overline{C}_f$ and $\overline{R}_\theta$ are seen to be well predicted.*

4.2 SUPERSONIC - CONSTANT PRESSURE

Figure 6 shows typical comparisons of the theory and experimental constant pressure adiabatic data of Coles.[5] The agreement is remarkably good considering that no recourse was made to the high speed experiment for the initial conditions used in the calculations.

4.3 SUPERSONIC - VARIABLE PRESSURE

There appears to be a remarkable lack of consistent detailed experimentation on two-dimensional, supersonic turbulent boundary layers in variable pressure that do not involve separation.

We have chosen the shock-wave impingement experiments of Hakkinen[15] at Mach 1.5 to model theoretically. The predicted shear stress is in agreement with the experimental values (Figure 7) and the characteristics of the equivalent low speed flow are shown in Figure 8.

In order to explore the theoretical predictions at higher Mach number, we have taken a gradually increasing pressure $C_p = C_p(R_x)$ at various Mach numbers. The results shown in Figure 9 show an interesting and somewhat surprising reversal of the behavior of shear stress with increasing Mach number.

---

*$\overline{H}$ was found to be in good agreement with the experiment beyond $\overline{R_x} \cong 2 \times 10^6$, but there exists a discrepancy at lower Reynolds numbers which is believed due to the neglect of the laminar sublayer in the low speed integral formulation. This is not expected to be an important limitation for most high speed turbulent boundary layer situations.

This result as well as the theory in general must, of course, be verified by future experimentation.

4.4 BOUNDARY LAYER SEPARATION

We find that in the limit as $\bar{f} \to 0$, the low speed formulation predicts that $(\bar{\theta}/\bar{u}_e)\,(d\bar{u}_e/d\bar{x}) = -\,4.05 \times 10^{-3}$.

By recalling equation (12) and noting that

$$\frac{\bar{C}_f}{C_f} = \frac{R_\theta}{\bar{R}_\theta} = \frac{1}{\sigma'\dfrac{T_e}{\bar{T}}} \to \frac{T_w}{T_\infty}$$

we find that, independent of the history of the turbulent boundary layer,

$$\left\{ \left(\frac{T_w}{T_e}\right)^2 \frac{\theta}{\rho_e u_e^2} \frac{dp}{dx} \right\}_{separation} = 4.05 \times 10^{-3}$$

The result is in good agreement (≈10%) with the empirical correlation of Zukoski[16] which represents a compilation of adiabatic wall, turbulent boundary layer separation data for $M_\infty = 2 \to 6$.

4.5 SUPERSONIC EQUILIBRIUM PROFILES

Clauser[17] has shown that by adjusting the pressure distribution such that

$$\frac{\bar{\delta}^*}{\bar{\tau}_w} \frac{d\bar{p}}{d\bar{x}} = \bar{\beta}_T = constant$$

the outer portion of low-speed turbulent boundary layer velocity profiles can be maintained in a state of "equilibrium", i.e., they are invariant in the sense that

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$$\frac{\bar{u}_e - \bar{u}}{\bar{u}_\tau} = F\left(\frac{\bar{y}}{\displaystyle\int_0^\delta \frac{\bar{u}_e - \bar{u}}{\bar{u}_\tau}\, d\bar{y}} \; ; \; \bar{\beta}_T \right)$$

By following the rules of the transformation and invoking the pressure gradient relationship (equation 12), we can generalize this concept to supersonic flows. The supersonic equivalent of Clauser's low speed equilibrium boundary layer is predicted to be of the form

$$\frac{u_e - u}{u_\tau} \cdot \alpha = F\left\{ \frac{\displaystyle\int_0^y \frac{\rho}{\rho_e}\, dy}{\displaystyle\int_0^\delta \frac{u_e - u}{u_\tau} \frac{\rho}{\rho_e}\, dy \cdot \alpha} \; ; \; \bar{\beta}_T \right\}$$

where

$$\alpha = \sqrt{1 + \langle f^2 \rangle\, m_e\, C_f/2} \qquad \left(\frac{T_w}{T_{o_\infty}} = 1 \right)$$

and

$$\bar{\beta}_T = (1 - m_e H) \frac{\delta^*}{\tau_w} \frac{dp}{dx}$$

The modeling outlined in the paper can be used to generate the required pressure distributions for any initial boundary layer and Mach number corresponding to a given experimental facility.

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5. CONCLUSIONS

Summarizing, we have

(1) found that the deviation of the high speed velocity profiles from the equivalent low speed profile occurs primarily in the inner region of the boundary layer, not in the wake.

(2) given evidence that the discrepancy is due to increased "dissipation" near the laminar sublayer in the compressible flow and concluded that, for not excessively high Mach numbers, this should not affect the application of the theory to obtain overall results.

(3) derived the equations which govern the scaling functions for a variable pressure in terms of the integral properties of the low speed turbulent boundary layer.

(4) set forth a complete formulation of the modeling of the supersonic turbulent boundary layer to its low speed equivalent and developed a computer program to implement it.

(5) applied the theory to a variety of supersonic turbulent boundary layer flows.

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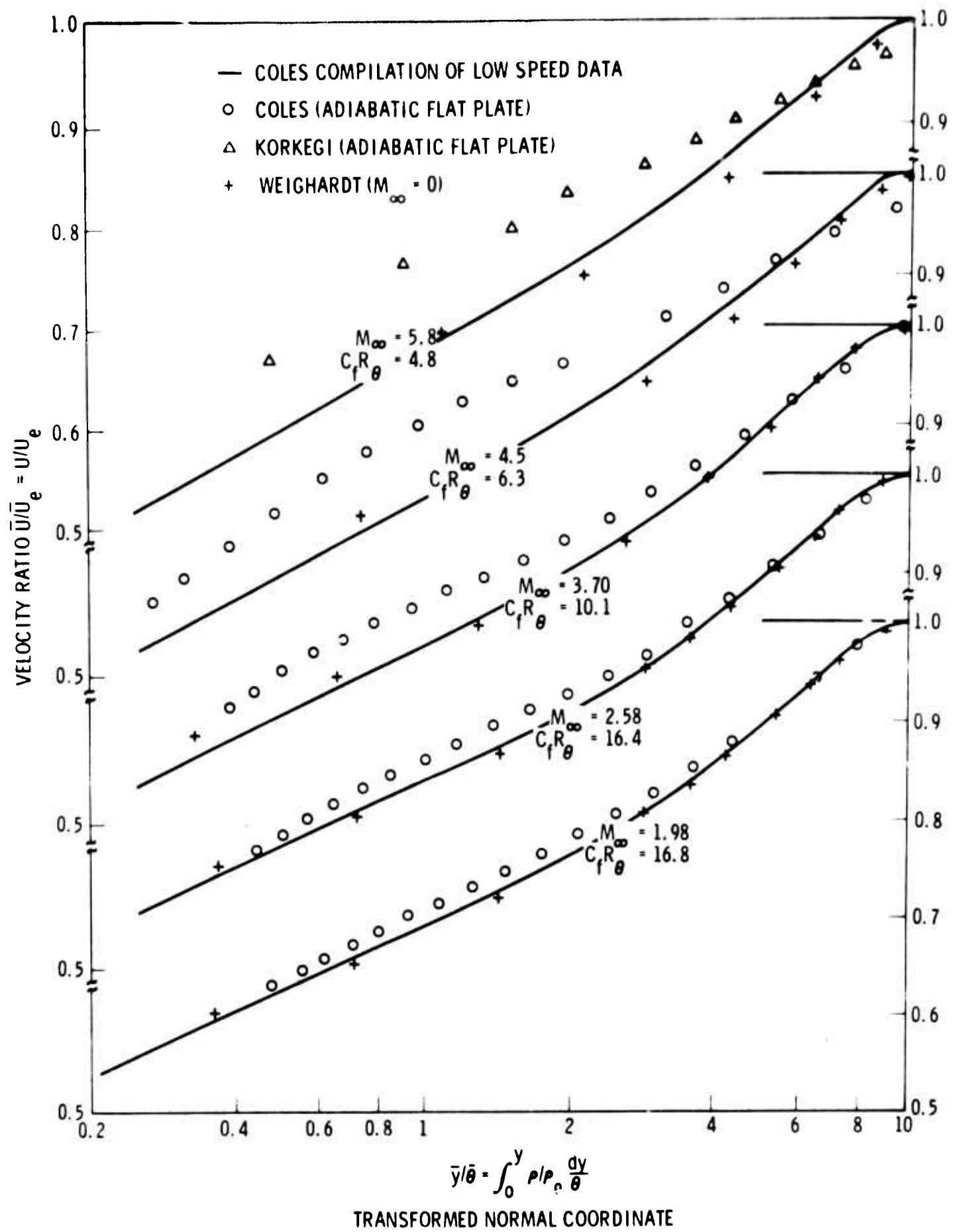19. Mellor, G. L. and Gibson, D. M., "Equilibrium Turbulent Boundary Layers," JFM, Vol. 24, 1966.

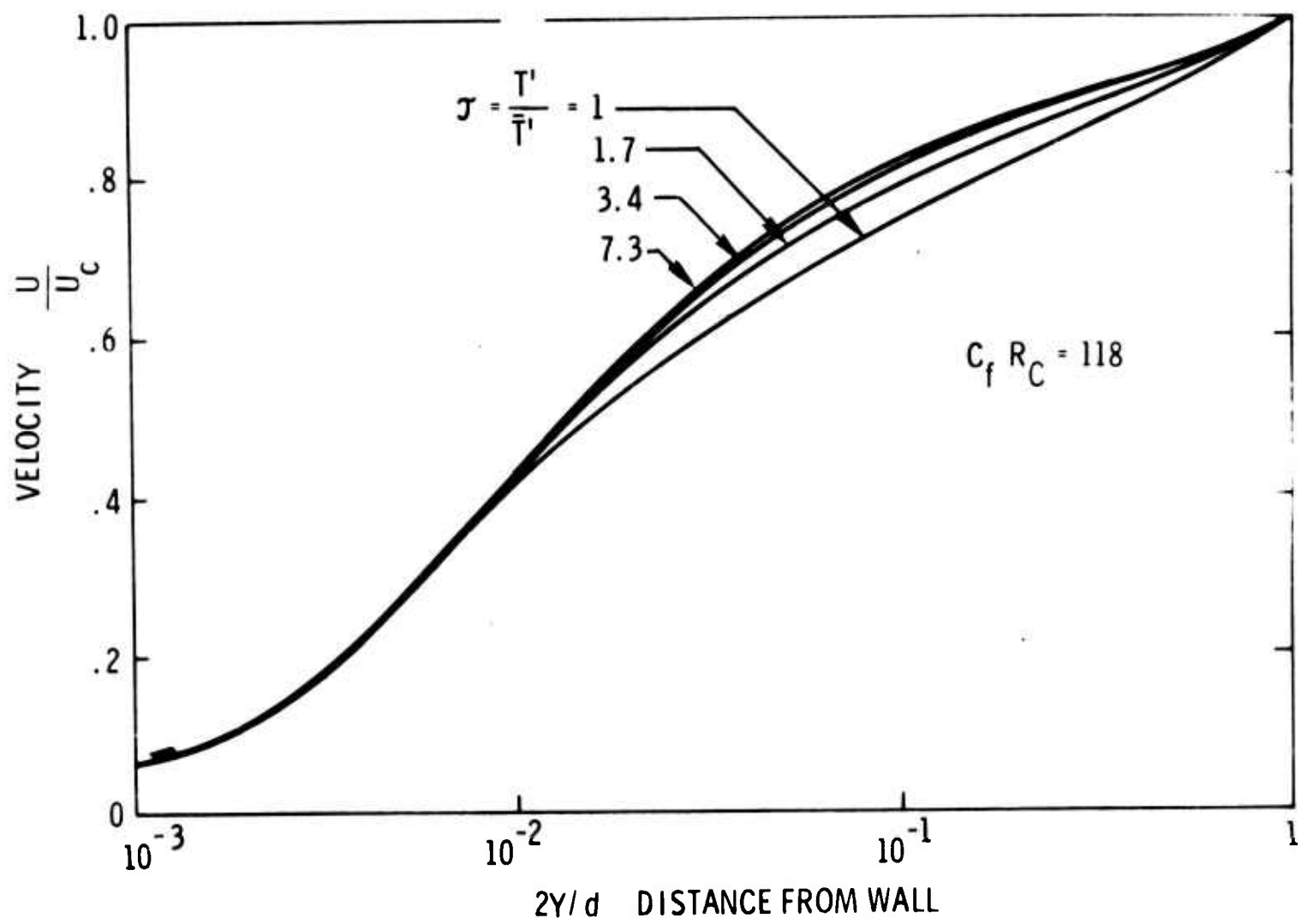Figure 1. Transformed High Speed Velocity Profiles.

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Figure 2. Effect of Arbitrarily Increasing Dissipation in Pipe Flow.
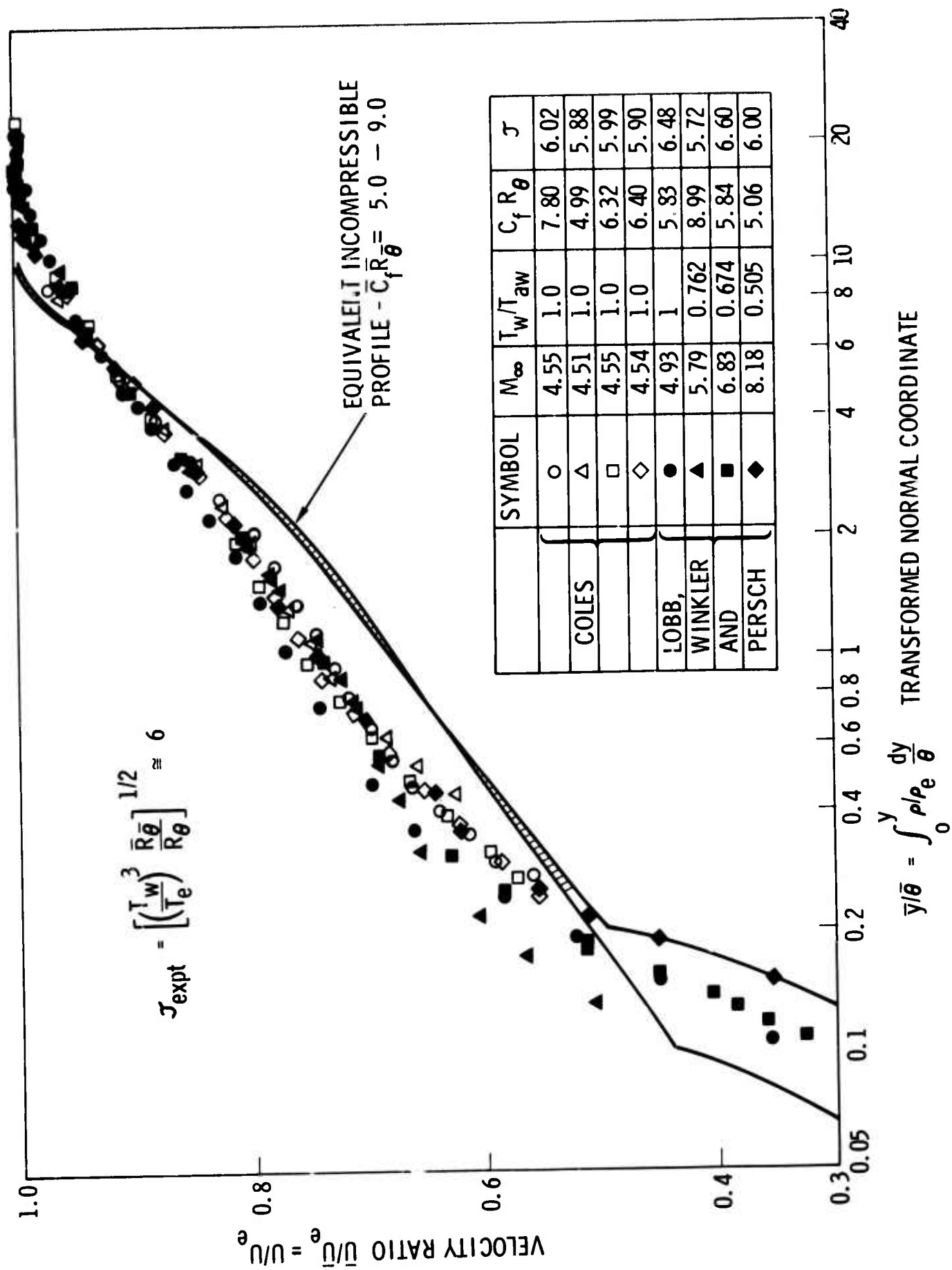
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EQUIVALENT INCOMPRESSIBLE
PROFILE $- \bar{C}_f \bar{R}_{\bar{\theta}} = 5.0 - 9.0$

	SYMBOL	M_∞	T_w/T_{aw}	$C_f R_\theta$	$\mathcal{J}$
COLES	○	4.55	1.0	7.80	6.02
	△	4.51	1.0	4.99	5.88
	□	4.55	1.0	6.32	5.99
	◇	4.54	1.0	6.40	5.90
LOBB,	●	4.93	1	5.33	6.48
WINKLER	▲	5.79	0.762	8.99	5.72
AND	■	6.83	0.674	5.84	6.60
PERSCH	◆	8.18	0.505	5.06	6.00

$$\mathcal{J}_{expt} = \left[\left(\frac{T_w}{T_e} \right)^3 \frac{\bar{R}_{\bar{\theta}}}{R_\theta} \right]^{1/2} \cong 6$$

$\bar{y}/\bar{\theta} = \int_0^y \frac{\rho/\rho_e}{\theta} \, dy$ TRANSFORMED NORMAL COORDINATE

VELOCITY RATIO $\bar{U}/\bar{U}_e = U/U_e$

Figure 3. Transformed High Speed Velocity Profiles $- \mathcal{J} \cong$ Constant.
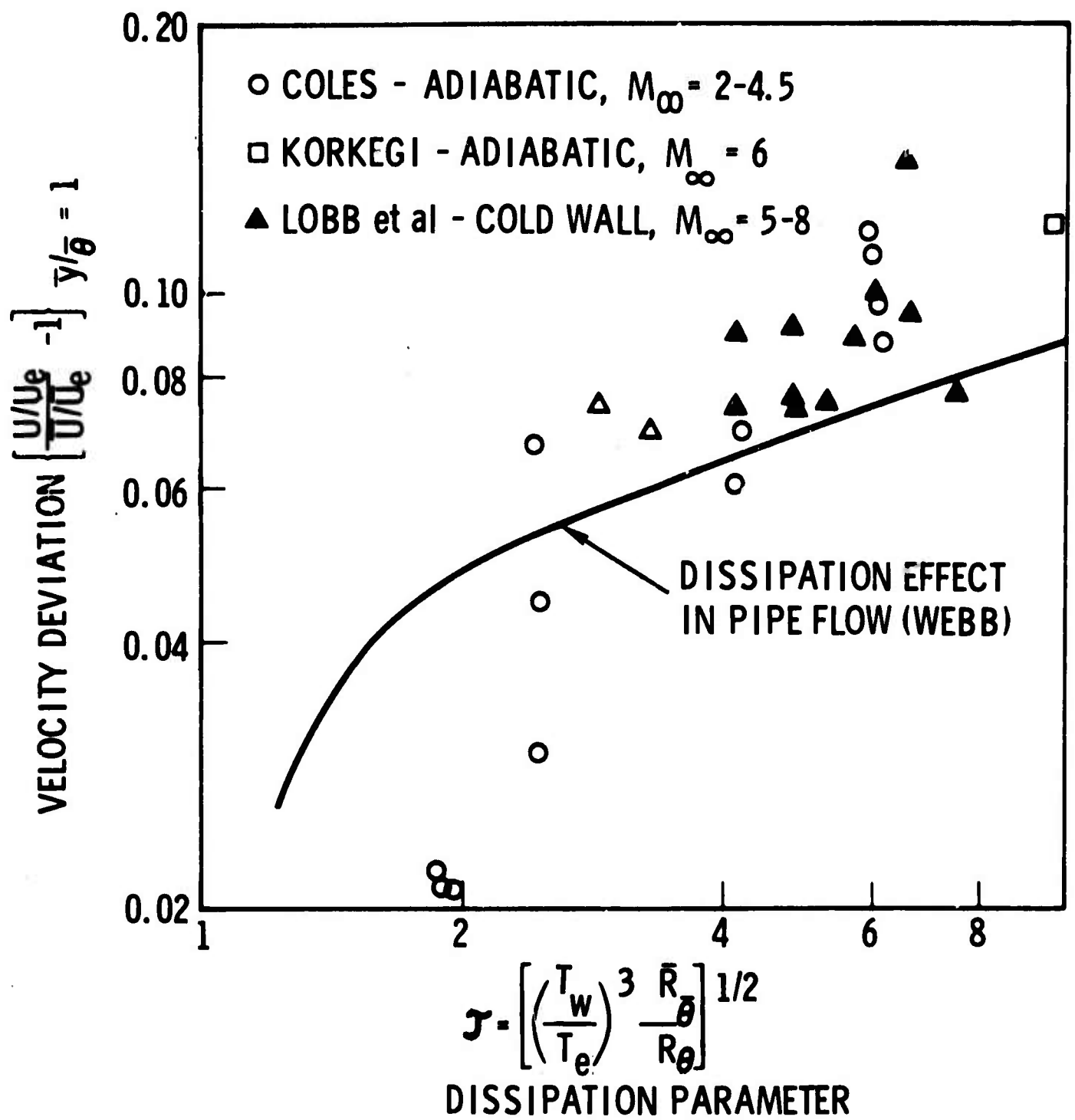
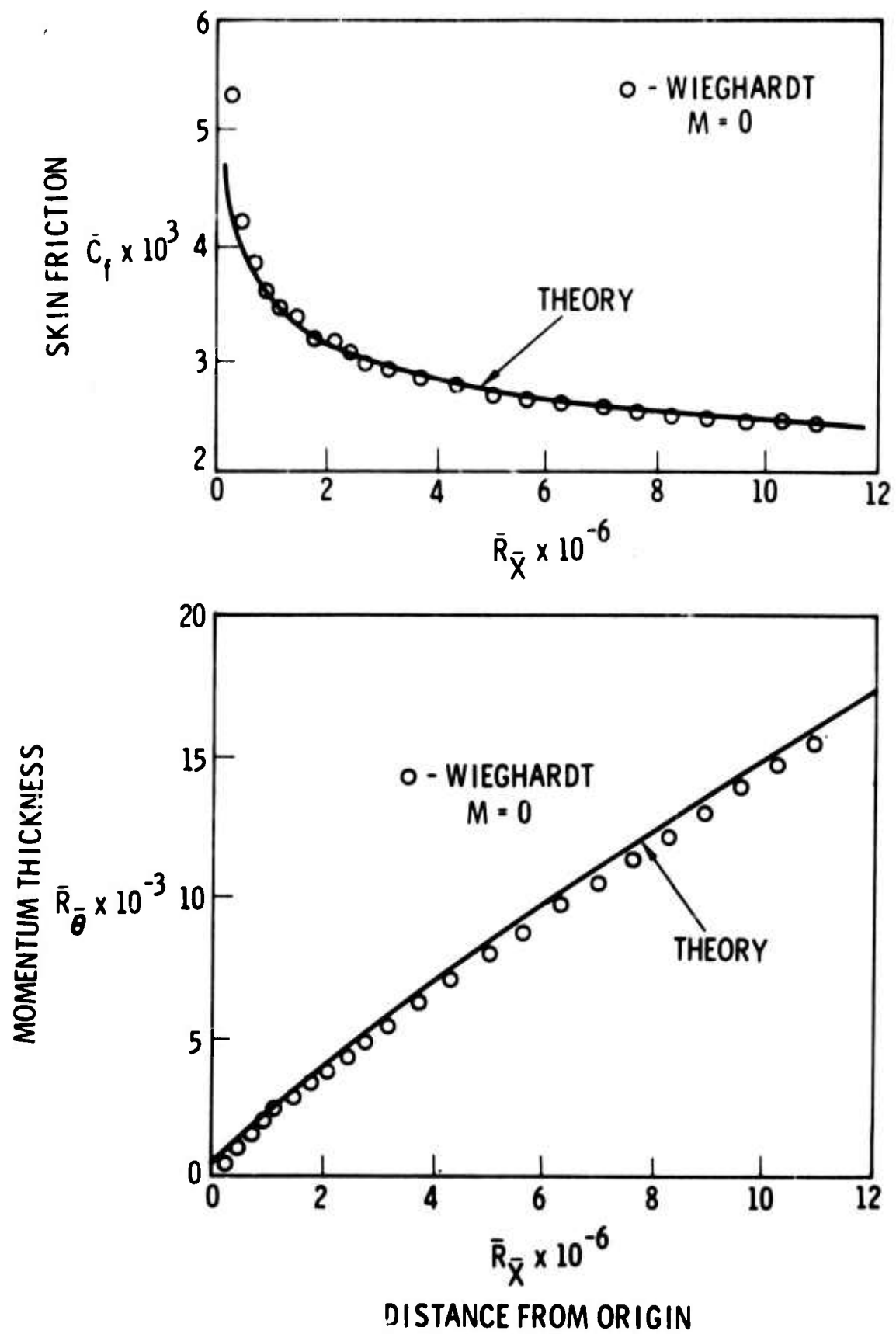Figure 4. Dissipation Effect.

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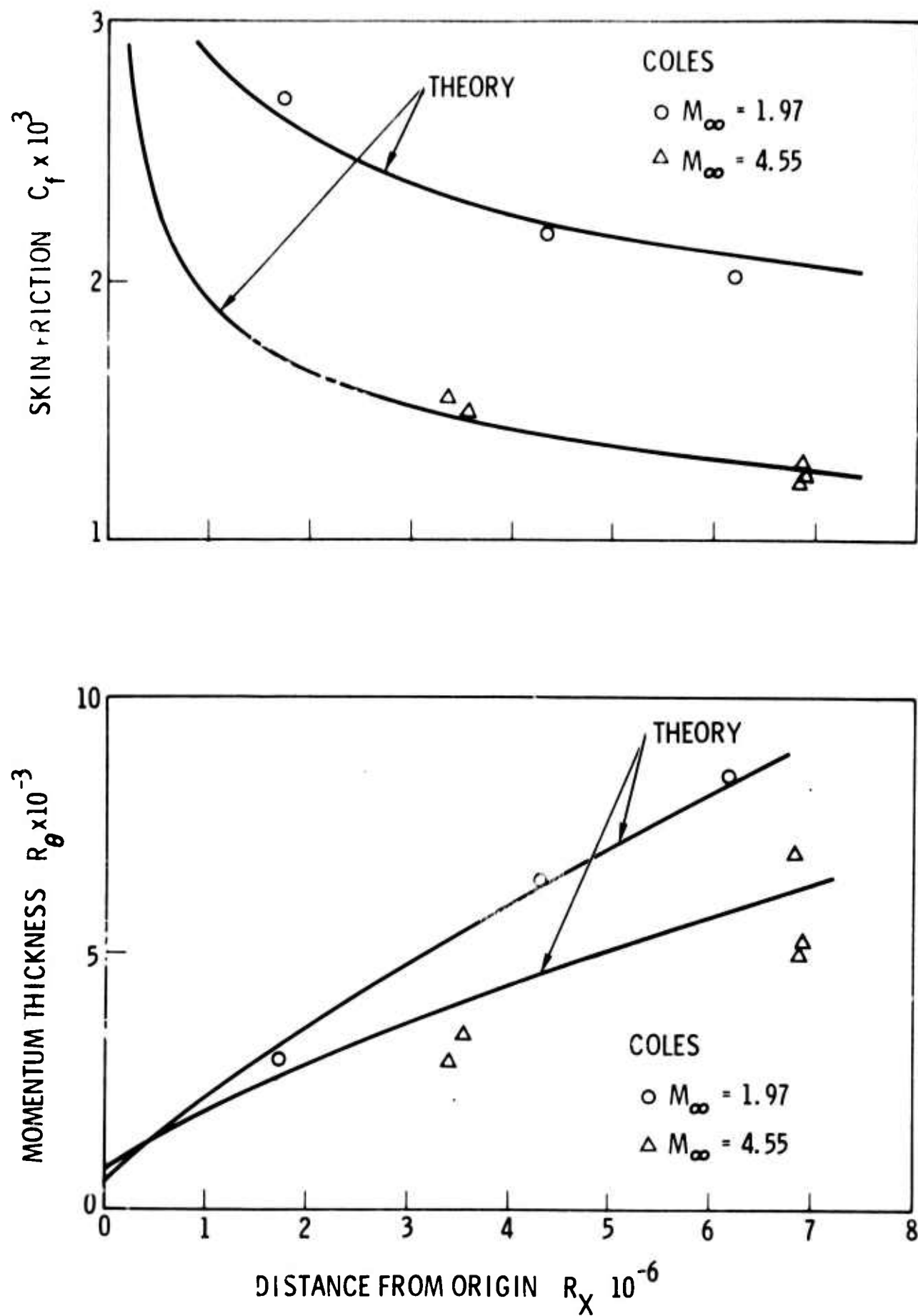Figure 5. Incompressible Integral Properties – Constant Pressure.

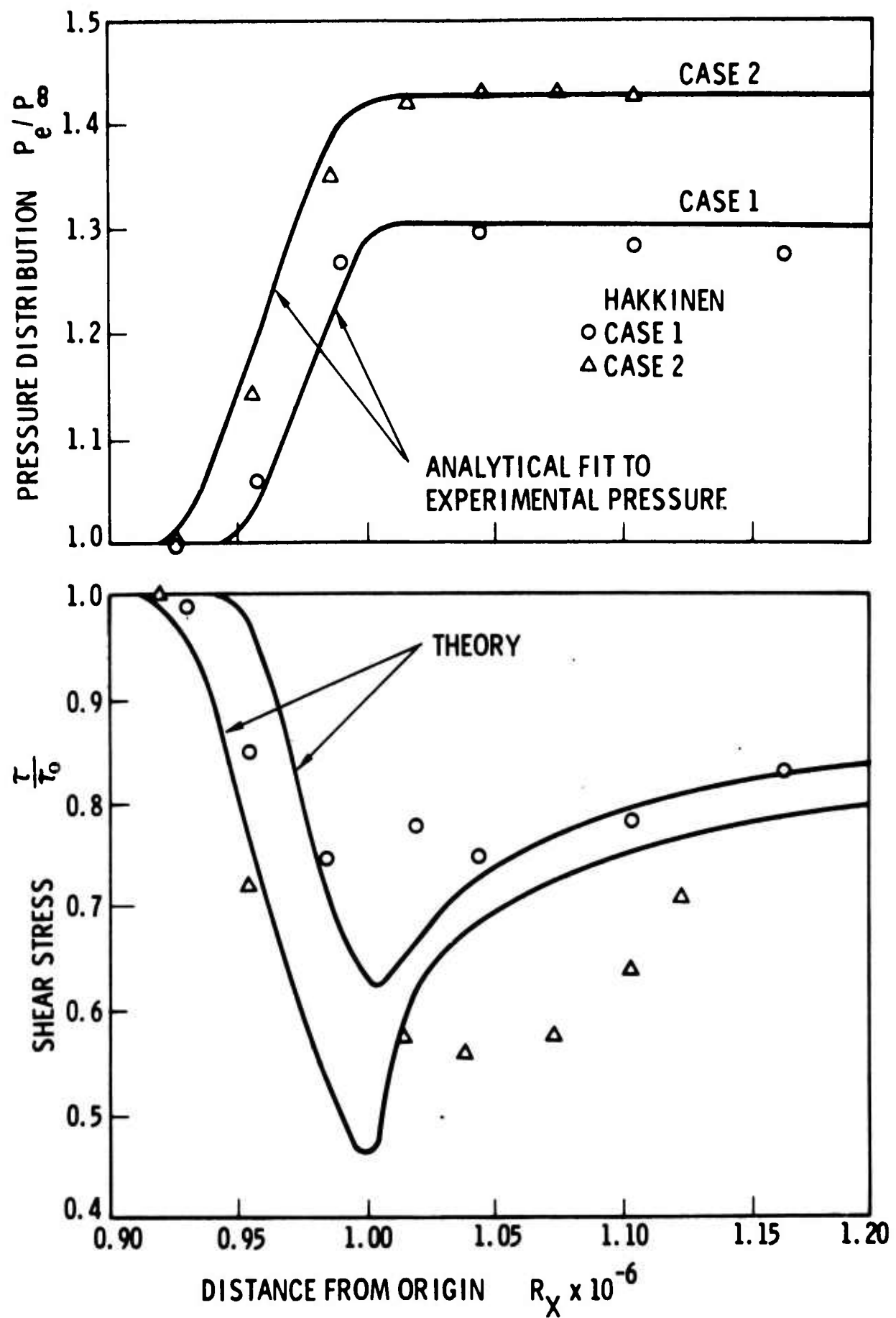Figure 6.   Supersonic Integral Properties – Constant Pressure.

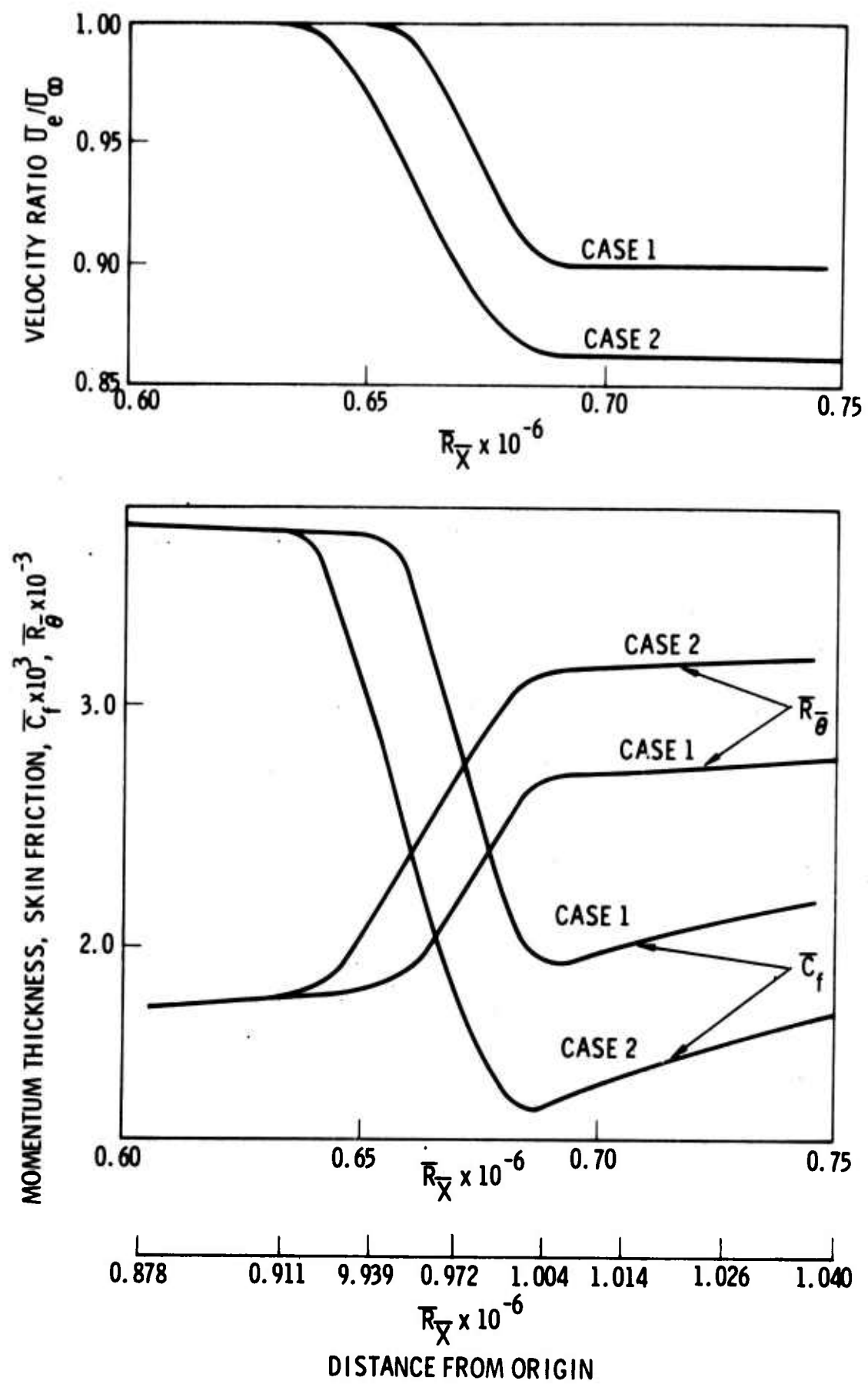Figure 7. Supersonic Integral Properties - Variable Pressure.

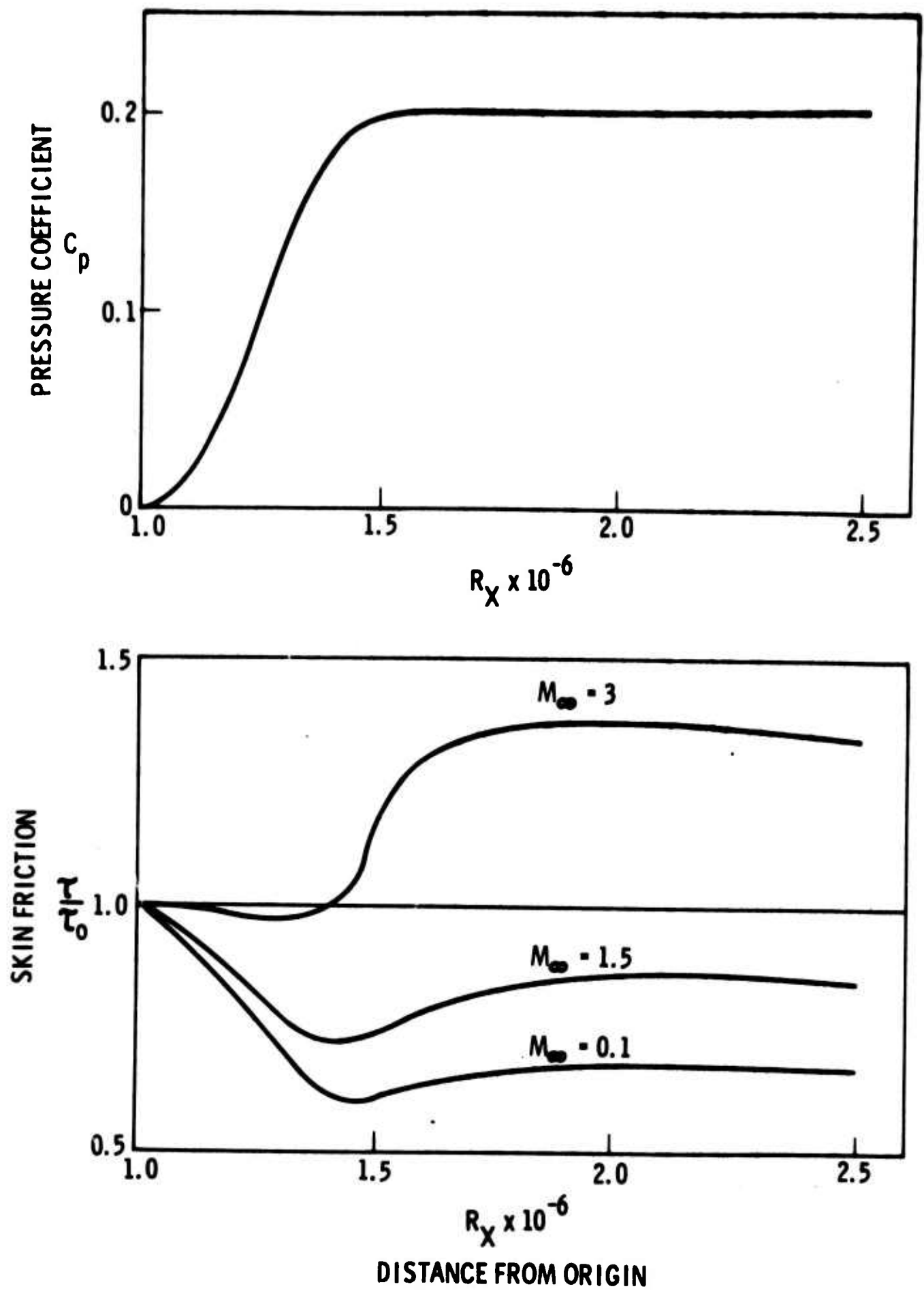Figure 8. Low Speed Equivalent of Hakkinen's Supersonic Experiment.

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Figure 9. Mach Number Effect on Shear Stress in Region of Gradually Increasing Pressure.

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APPENDIX
LOW SPEED INTEGRAL FORMULATION

The formulation [see Alber[18]] used consists of the integrated form of the momentum equation

$$\bar{H} \frac{d\bar{R}_\delta^*}{d\bar{R}_x} + \bar{R}_\delta^* \frac{\partial \bar{H}}{\partial \tilde{\pi}} \frac{d\tilde{\pi}}{d\bar{R}_x} + \bar{R}_\delta^* \frac{\partial \bar{H}}{\partial \bar{f}} \frac{d\bar{f}}{d\bar{R}_x} = \bar{f}^2 - (2H + 1) \frac{\bar{R}_\delta^*}{\bar{u}_e} \frac{d\bar{u}_e}{d\bar{R}_x} \tag{24}$$

and its first moment

$$\bar{J} \frac{d\bar{R}_\delta^*}{d\bar{R}_x} + \bar{R}^* \frac{\partial \bar{J}}{\partial \tilde{\pi}} \frac{d\tilde{\pi}}{d\bar{R}_x} + \bar{R}_\delta^* \frac{\partial \bar{J}}{\partial \bar{f}} \frac{d\bar{f}}{d\bar{R}_x} = \bar{C}_D - 3\bar{J} \frac{\bar{R}_\delta^*}{\bar{u}_e} \frac{d\bar{u}_e}{d\bar{R}_x} \tag{25}$$

and the differential form of the skin friction law

$$\frac{d\bar{R}_\delta^*}{d\bar{R}_x} + \bar{R}_\delta^* [P] \frac{d\tilde{\pi}}{d\bar{R}_x} + \bar{R}_\delta^* [Q] \frac{d\bar{f}}{d\bar{R}_x} = - \frac{\bar{R}_\delta^*}{\bar{u}_e} \frac{d\bar{u}_e}{d\bar{R}_x} \tag{26}$$

where $\tilde{\pi}$ comes from Coles'[1] analytical form of the turbulent boundary layer

$$\frac{\bar{u}}{\bar{u}_\tau} = \frac{1}{k} \ln \frac{\bar{y} \, \bar{u}_\tau}{\bar{\nu}} + C + \frac{\tilde{\pi}}{k} W(\bar{y}/\bar{\delta})$$

k = .40 (Karman's constant)

C = 5.10 and $\bar{u}_\tau = \sqrt{\tau_w/\rho}$, and $\bar{H}$, $\bar{J}$, [P], [Q] and $\bar{C}_D$ are specified functions of $\tilde{\pi}$ and $\bar{f}$ (see Reference 18).

In order to arrive at the dependence of $\overline{C}_D$ on $\widetilde{\pi}$ and $\overline{f}$, it is necessary to link $\overline{\beta}_T = (\overline{\delta}^*/\overline{\tau}_w) (d\overline{p}/d\overline{x})$ to

$$\overline{G} = \frac{\displaystyle\int_0^{\delta} \left(\frac{\overline{u}_e - \overline{u}}{\overline{u}_\tau}\right)^2 d\overline{y}}{\displaystyle\int_0^{\delta} \left(\frac{\overline{u}_e - \overline{u}}{\overline{u}_\tau}\right) d\overline{y}}$$

and here we have modified Alber's formulation to agree with the numerical values predicted for equilibrium flows by Mellor and Gibson[19] for which the following form was chosen

$$\overline{\beta}_T = - .852 - .0596 \, \overline{G} + (\overline{G}/5.90)^2$$

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13 ABSTRACT

We have reviewed Coles' analytical development of the transformation and its application to the constant pressure turbulent boundary layer. The limitations of the theory noted by Baronti and Libby are clarified, and an attempt is made to explain the remaining discrepancies. The equations governing the scaling functions (which link the high speed boundary layer to an equivalent low speed flow) are derived in terms of the integral properties of the low speed flow for an arbitrary pressure distribution. The modeling is completed by coupling the pressure gradients of the two flows and predicting the behavior of the integral properties of the low speed flow. The resulting formulation predicts the integral properties of the high speed turbulent boundary layer as well as the required modeling to an equivalent low speed flow. Finally, the theory is applied to a variety of situations.

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